properties of complex geometric figures. The reader will have many occasions to see this.

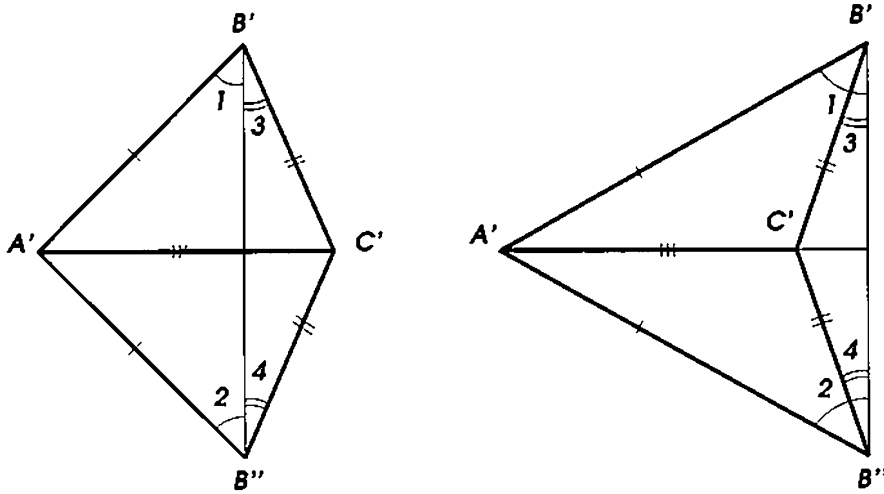

Figure 47

EXERCISES

75. Prove that a triangle that has two congruent angles is isosceles.

76. In a given triangle, an altitude is a bisector. Prove that the triangle is isosceles.

77. In a given triangle, an altitude is a median. Prove that the triangle is isosceles.

78. On each side of an equilateral triangle ABC, congruent segments AB', BC', and AC' are marked, and the points A', B', and C' are connected by lines. Prove that the triangle $A'B'C'$ is also equilateral.

79. Suppose that an angle, its bisector, and one side of this angle in one triangle are respectively congruent to an angle, its bisector, and one side of this angle in another triangle. Prove that such triangles are congruent.

80. Prove that if two sides and the median drawn to the first of them in one triangle are respectively congruent to two sides and the median drawn to the first of them in another triangle, then such triangles are congruent.

81. Give an example of two non-congruent triangles such that two sides and one angle of one triangle are respectively congruent to two sides and one angle of the other triangle.

*82.** On one side of an angle A, the segments AB and AC are marked, and on the other side the segments $AB' = AB$ and $AC' = AC$. Prove that the lines BC' and $B'C$ meet on the bisector of the angle A.

83. Derive from the previous problem a method of constructing the bisector using straightedge and compass.

**84.* Prove that in a convex pentagon: (a) if all sides are congruent, and all diagonals are congruent, then all interior angles are congruent, and (b) if all sides are congruent, and all interior angles are congruent, then all diagonals are congruent.

**85.* Is this true that in a convex polygon, if all diagonals are congruent, and all interior angles are congruent, then all sides are congruent?

7 Inequalities in triangles

41. Exterior angles. The angle supplementary to an angle of a triangle (or polygon) is called an **exterior angle** of this triangle (polygon).

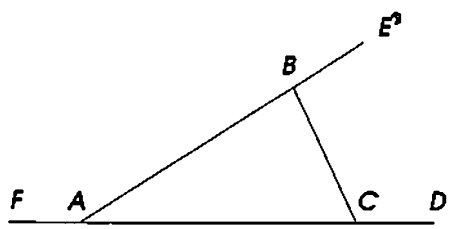

Figure 48

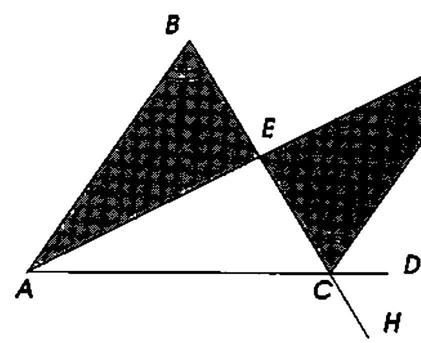

Figure 49

For instance (Figure 48), $\angle BCD$, $\angle CBE$, $\angle BAF$ are exterior angles of the triangle ABC. In contrast with the exterior angles, the angles of the triangle (polygon) are sometimes called **interior**.

For each interior angle of a triangle (or polygon), one can construct two exterior angles (by extending one or the other side of the angle). Such two exterior angles are congruent since they are vertical.

42. Theorem. ***An exterior angle of a triangle is greater than each interior angle not supplementary to it.***

For example, let us prove that the exterior angle BCD of $\triangle ABC$ (Figure 49) is greater than each of the interior angles A and B not supplementary to it.

Through the midpoint E of the side BC, draw the median AE and on the continuation of the median mark the segment EF congruent to AE. The point F will obviously lie in the interior of the